



# Relativized Succinct Arguments in the ROM Do Not Exist

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EPFL



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Succinct non-interactive arguments

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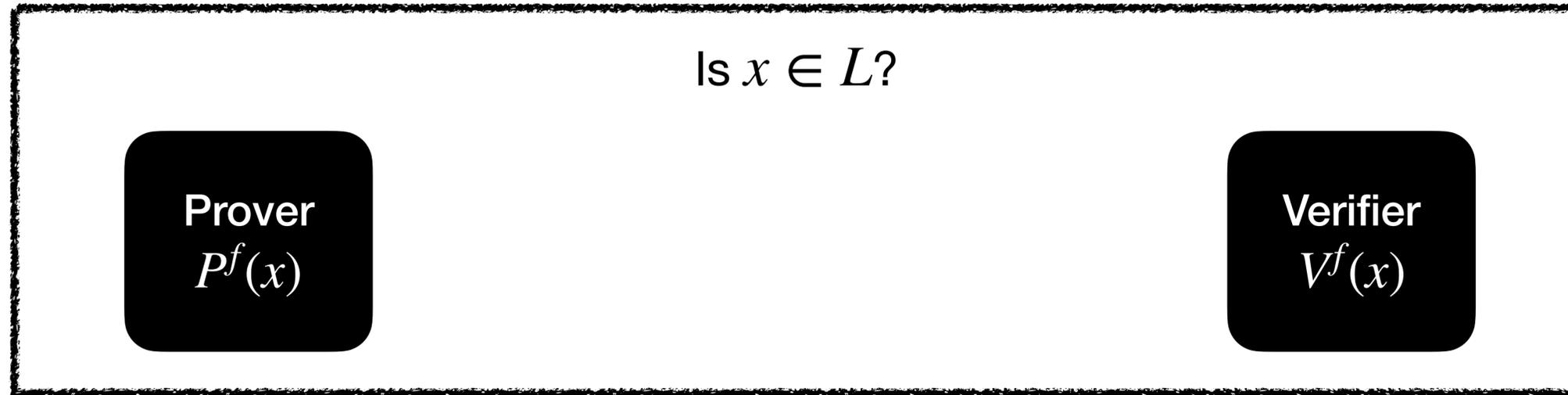


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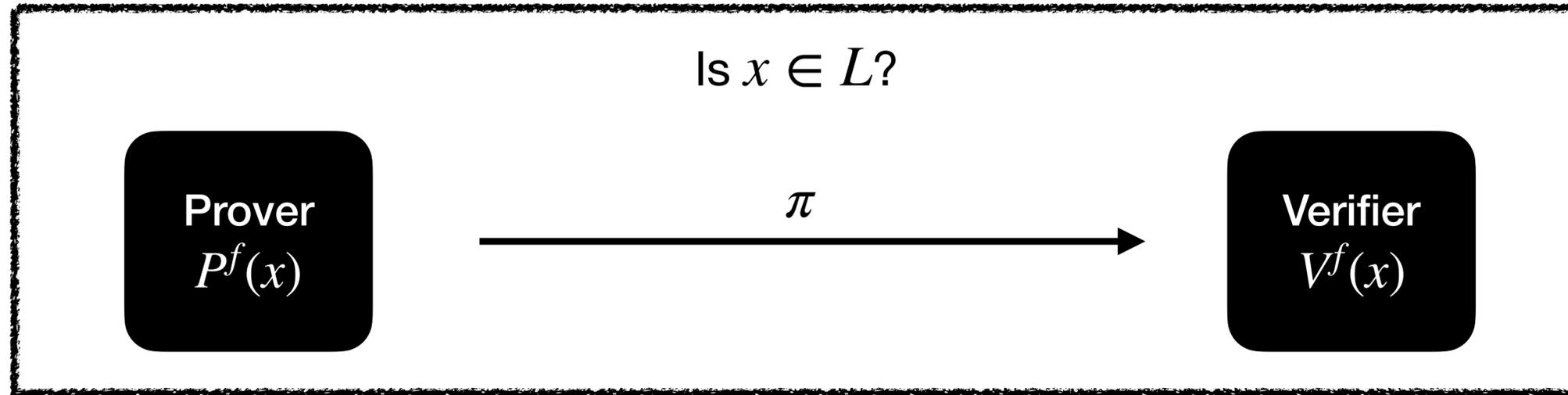


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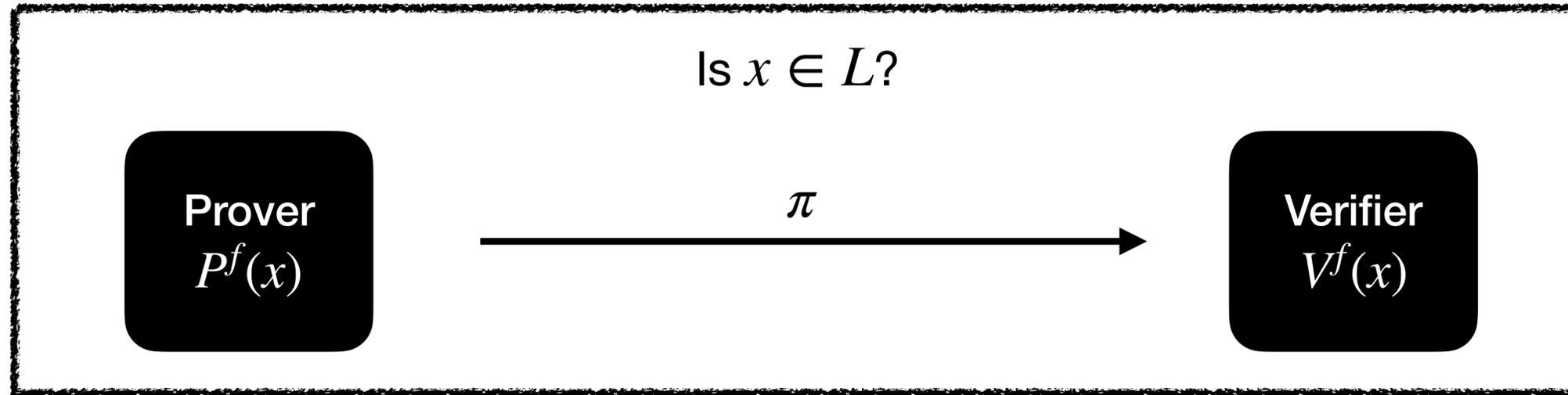


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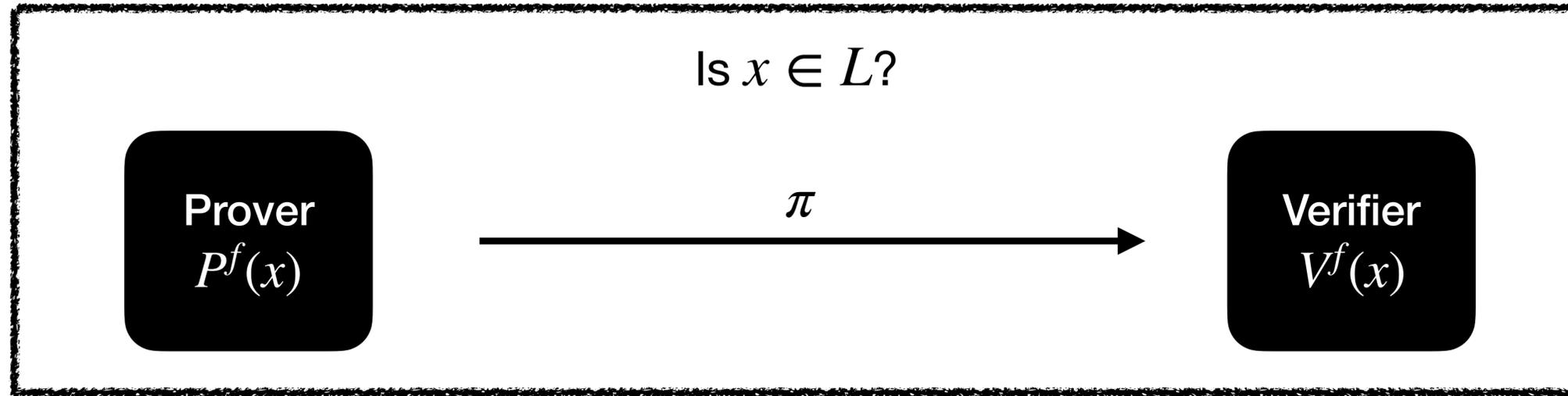
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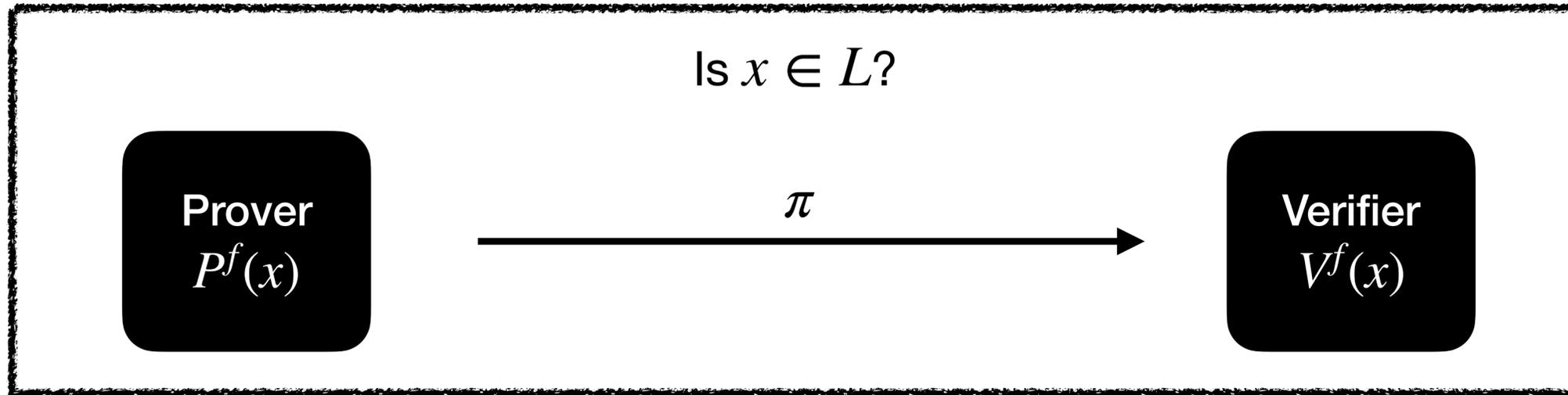
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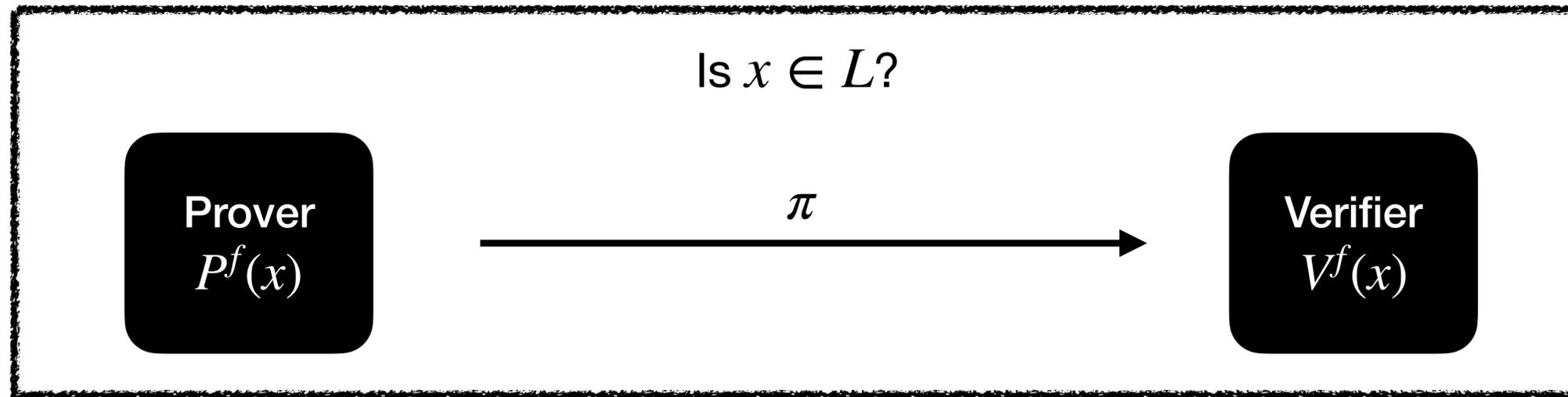
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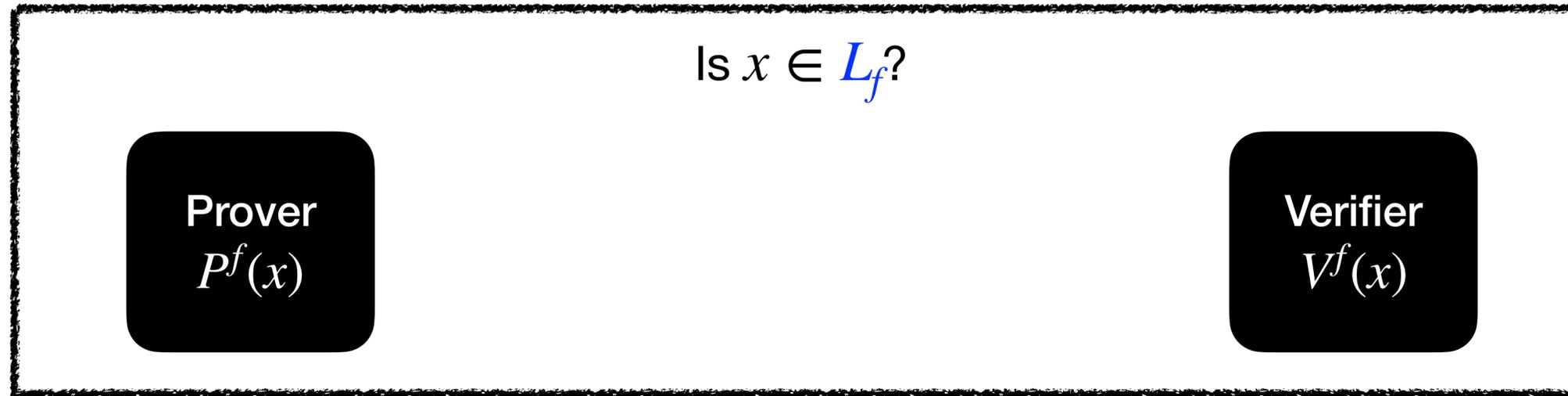


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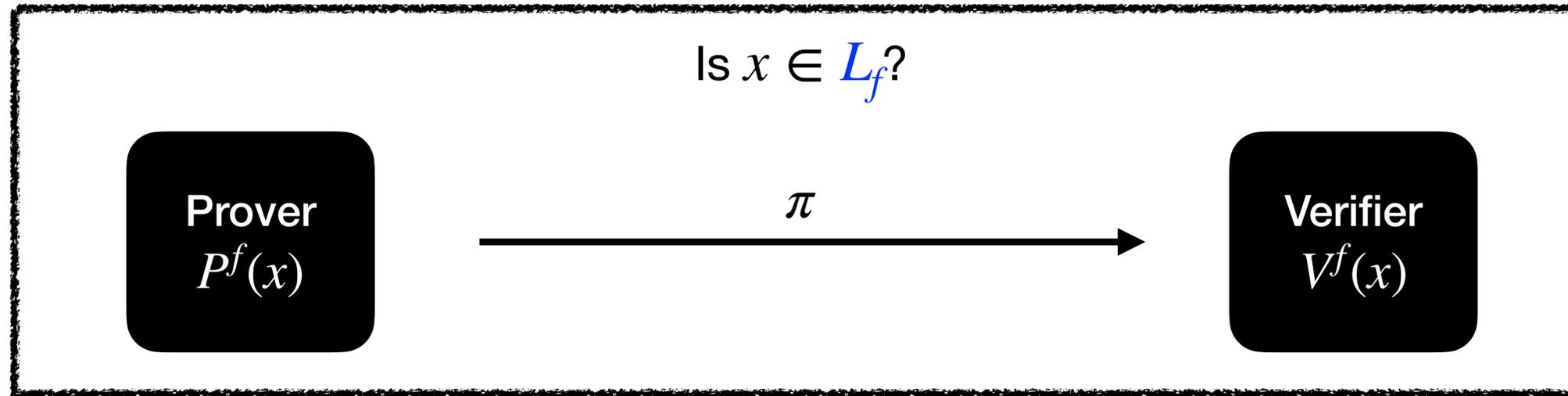


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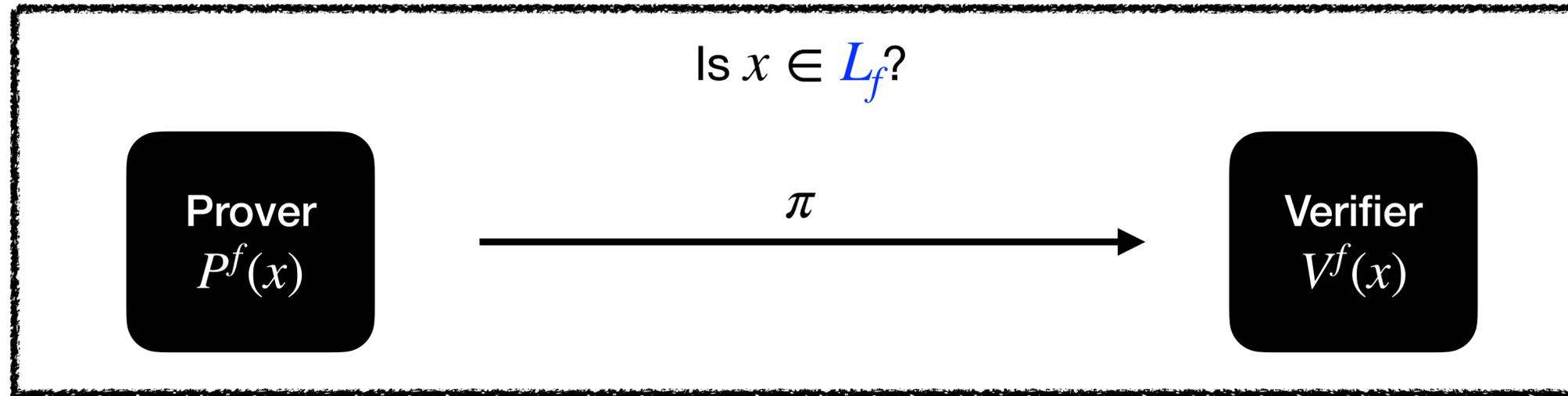


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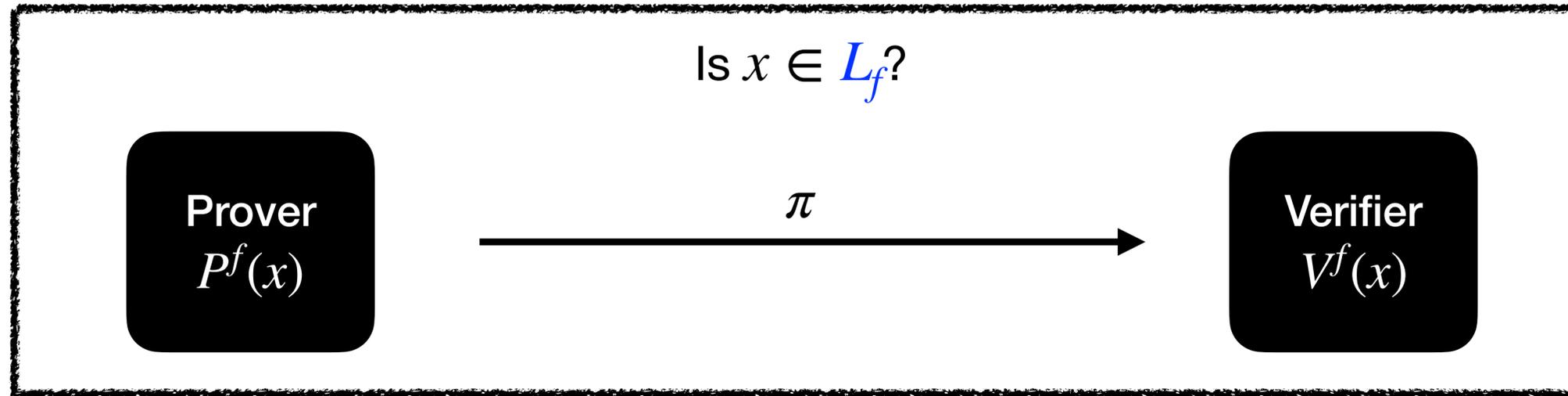
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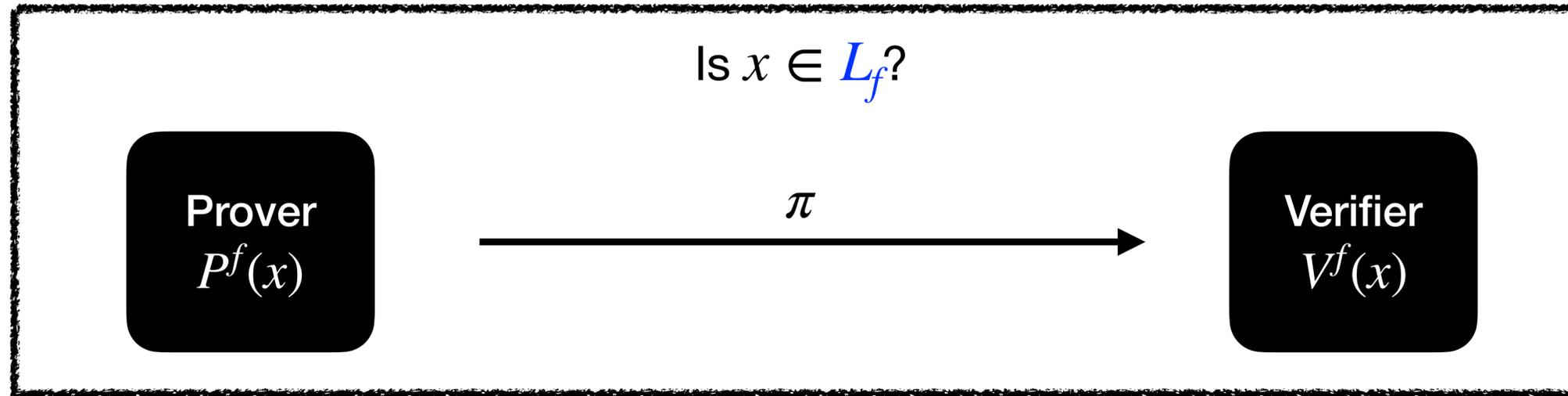
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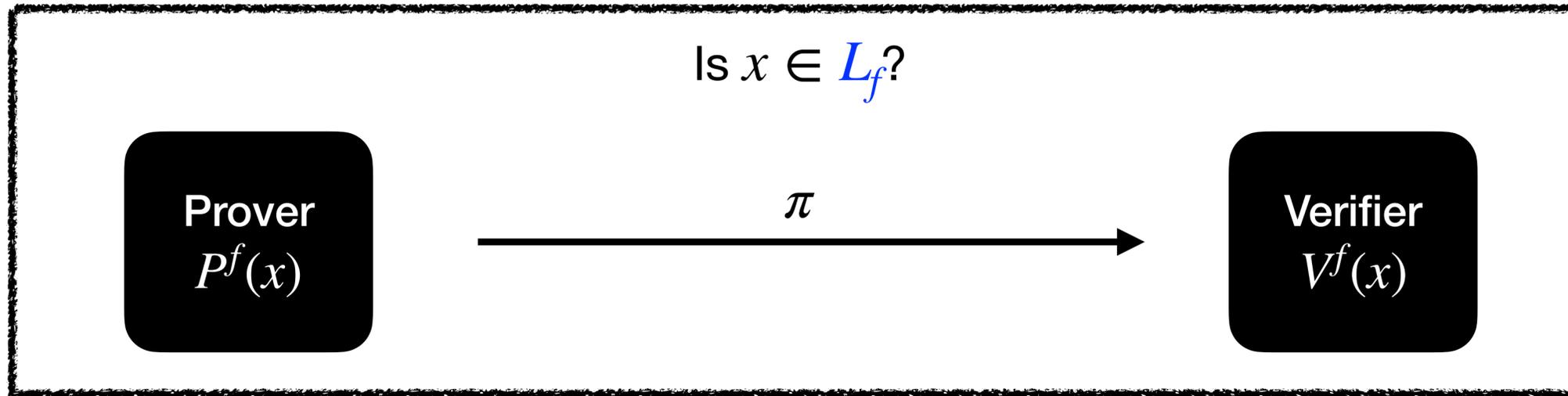
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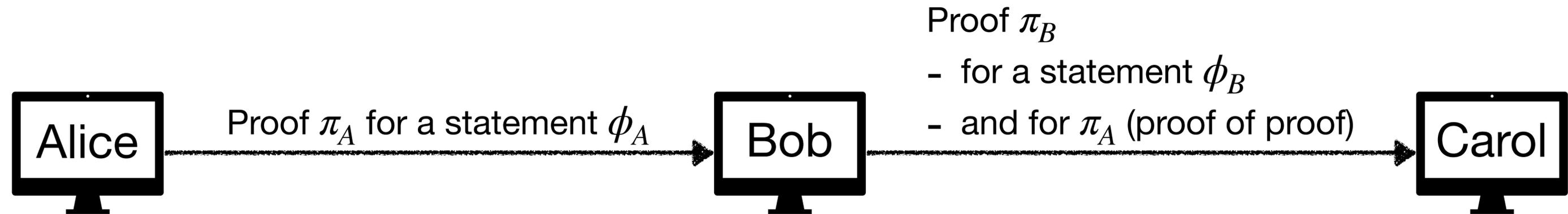
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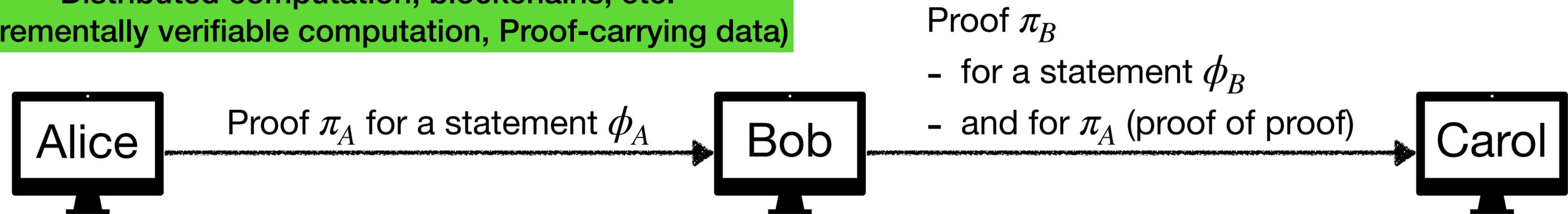
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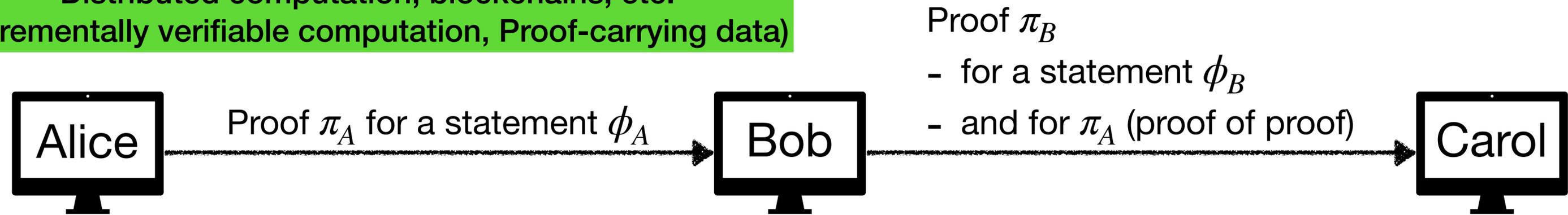
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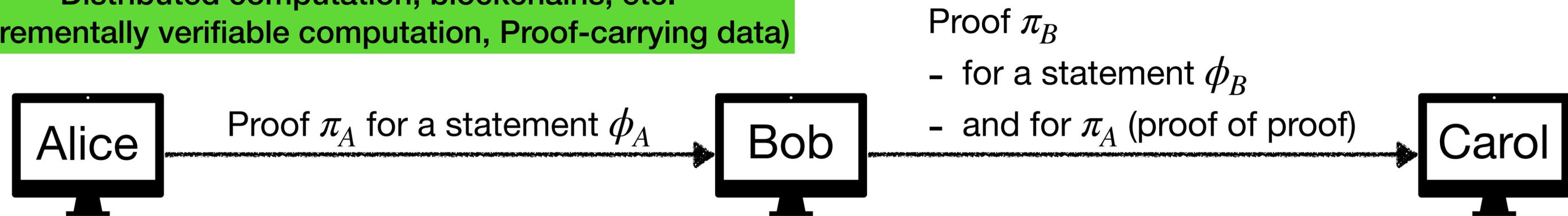


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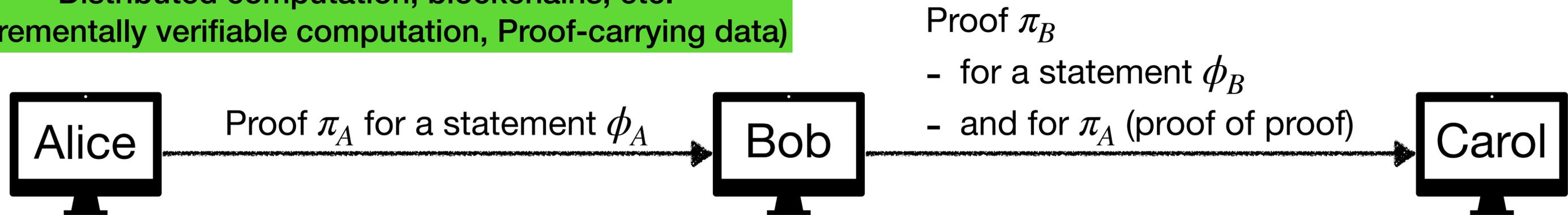
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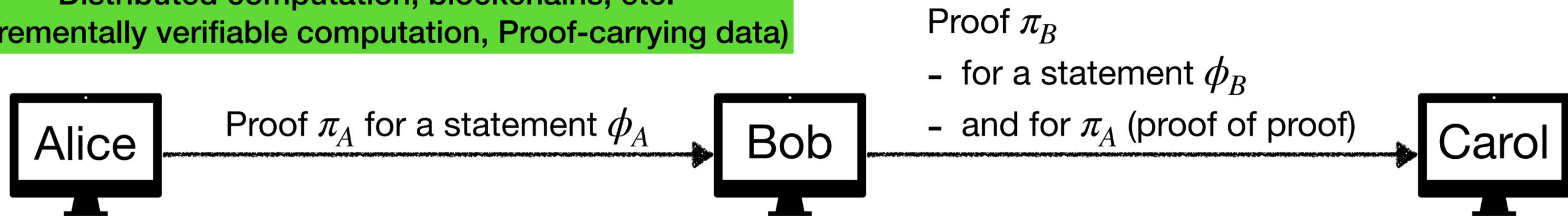
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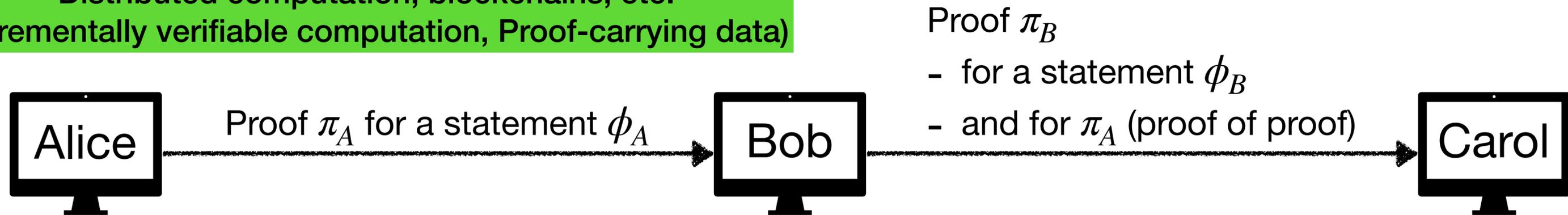
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Existence of IVC/PCD in the ROM still remains open.

**Corollary.** Relativized IVC/PCD does not exist in the ROM!

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Relativized arguments in the random oracle model do not exist.

verifier query complexity to the RO

Trivial Baseline 1.  $\text{DTIME}^{\mathcal{O}}[t] \subseteq \text{ARG}^{\mathcal{O}}[\overset{\uparrow}{\text{vq}} = t]$ .

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## Note.

- The results hold for SNARGs secure against query-bounded and time-bounded adversaries.
- Similar results hold for interactive arguments.

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Easy to learn/predict

Hard to learn/predict



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Structured Oracle

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Easy to construct relativized SNARGs:  
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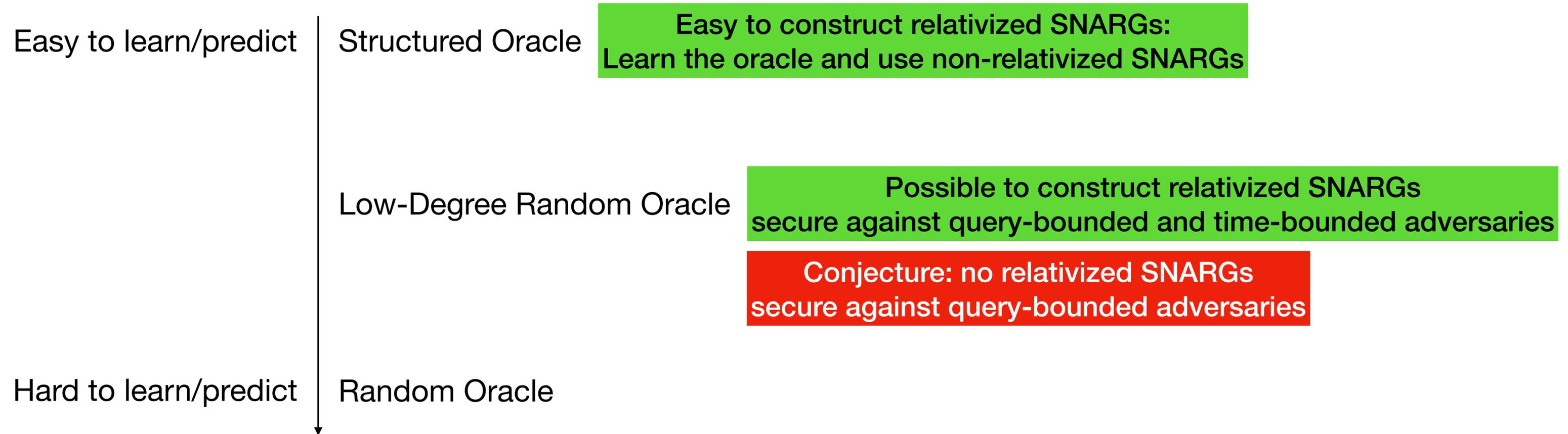
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Conjecture: no relativized SNARGs  
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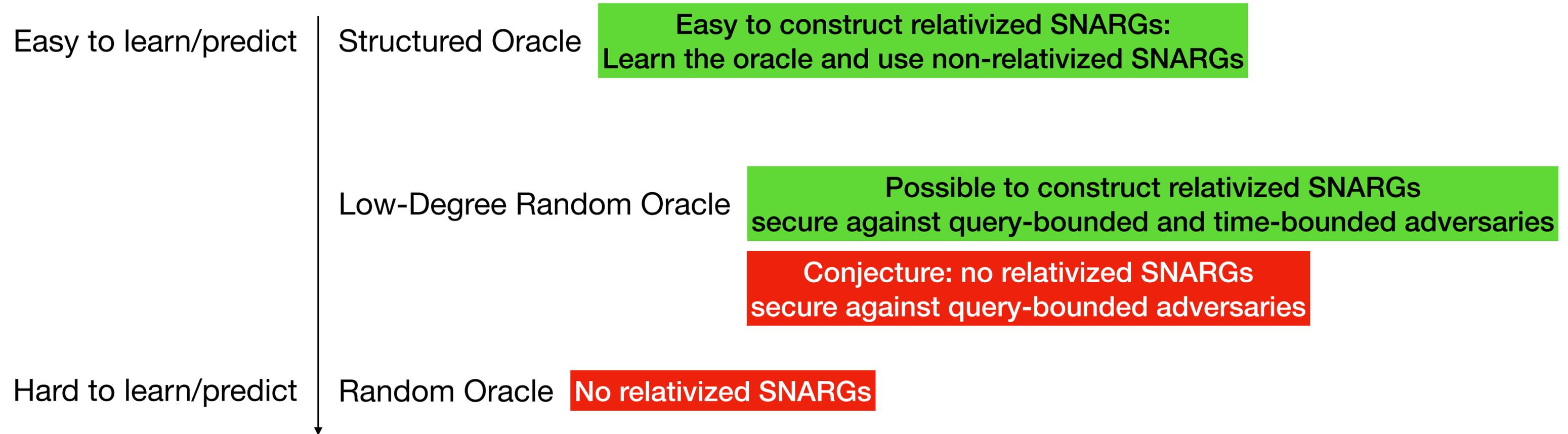
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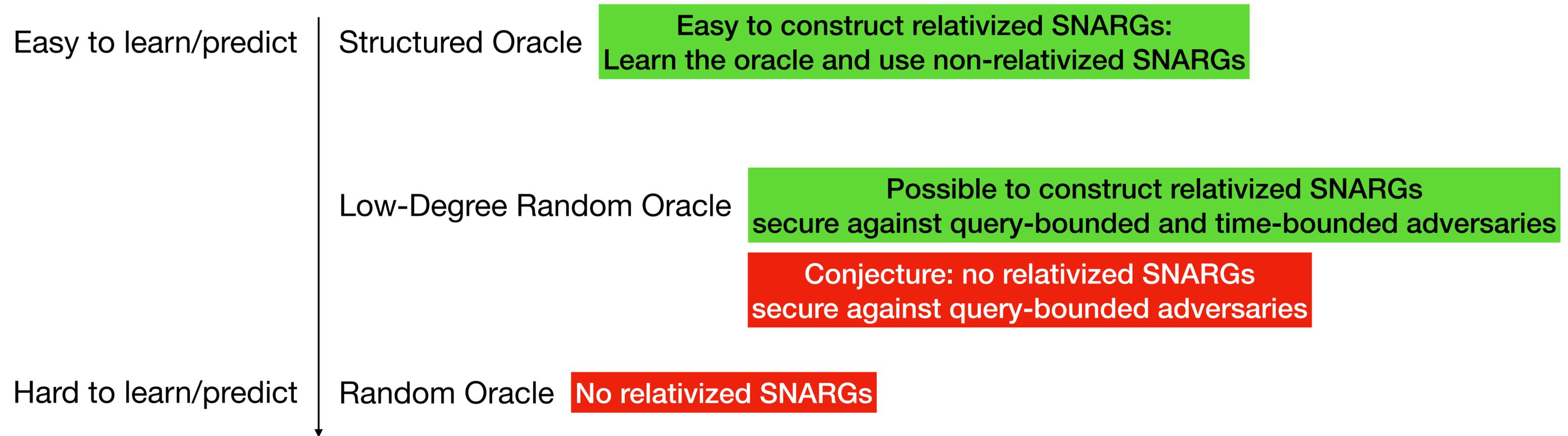
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## Open problem.

Give a sufficient and necessary condition for an oracle that separates  $DTIME/NTIME$  and relativized arguments.

# Insights into Fiat-Shamir

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Interactive protocol  
in the standard model

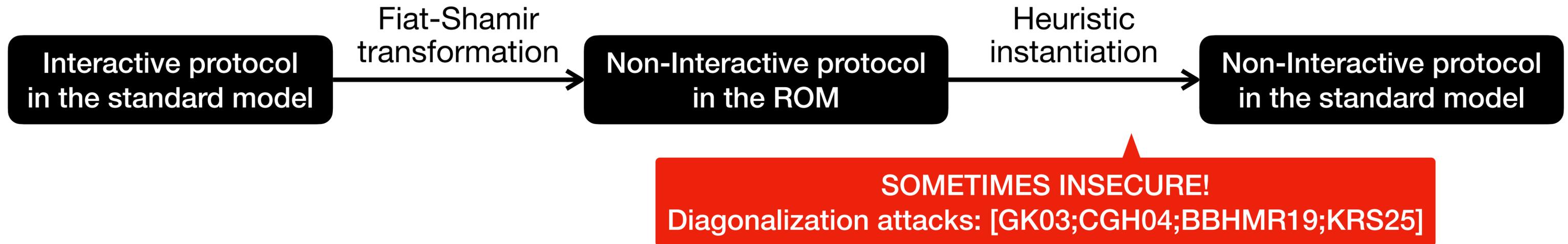
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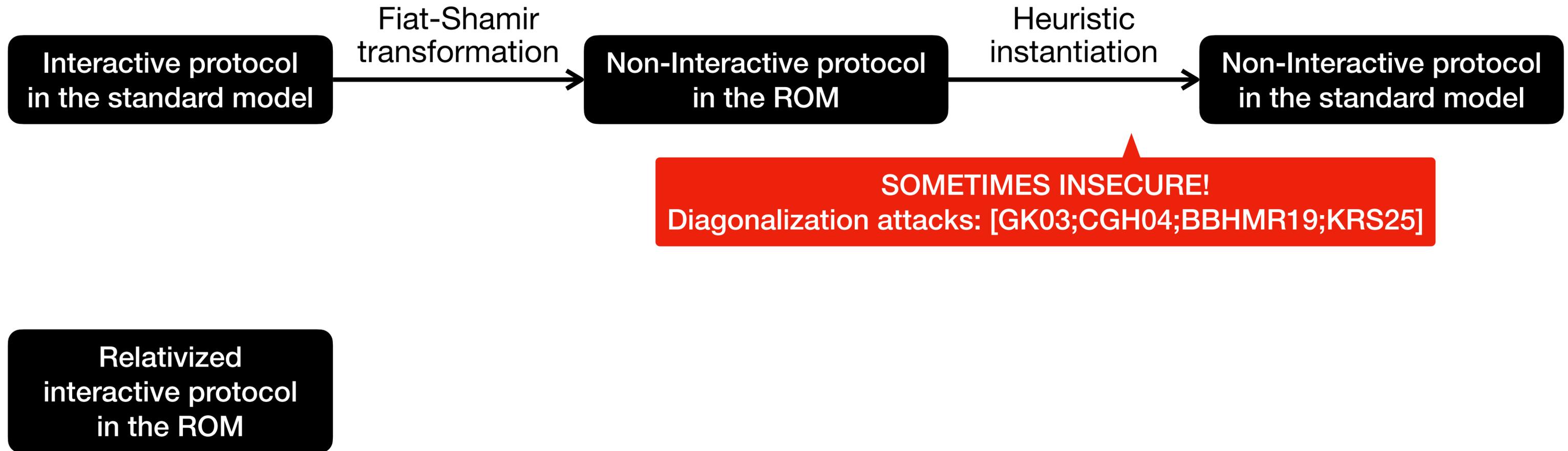
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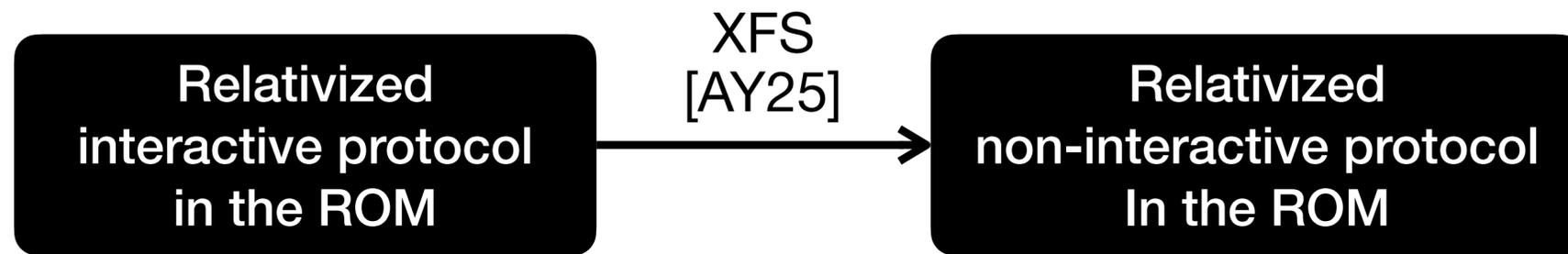
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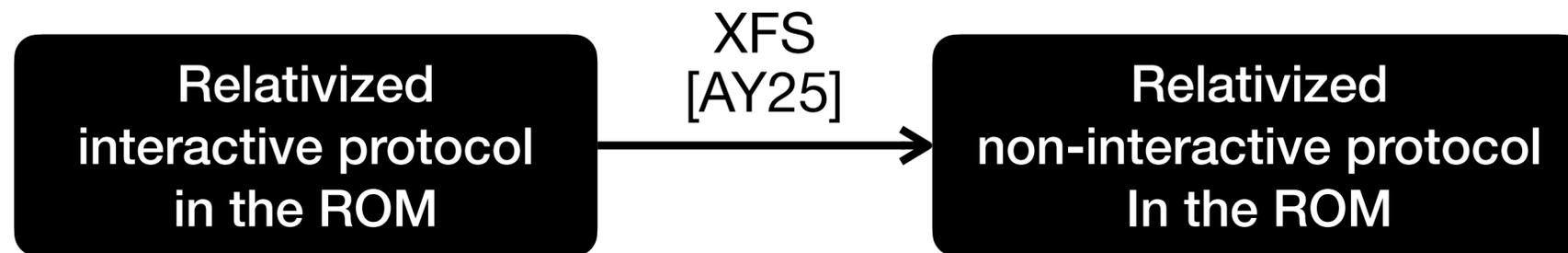
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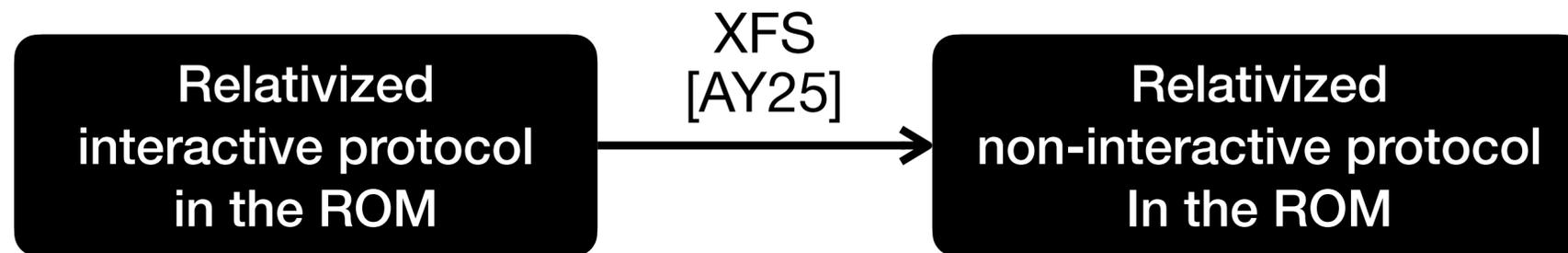


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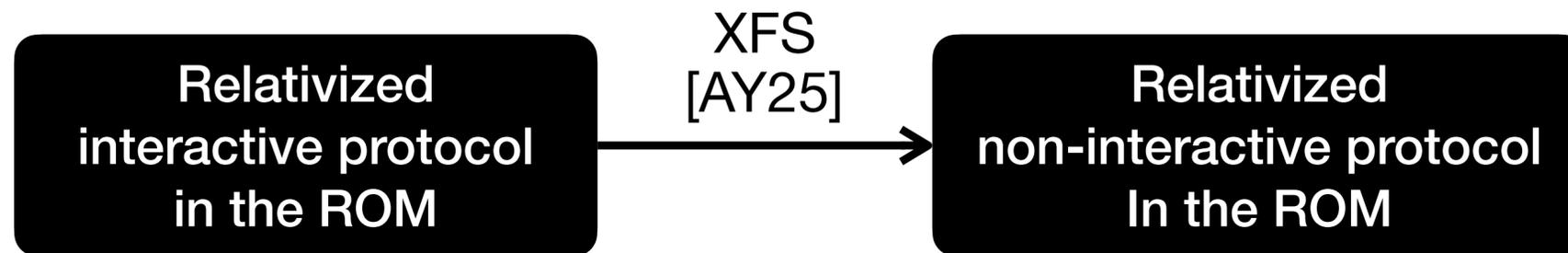


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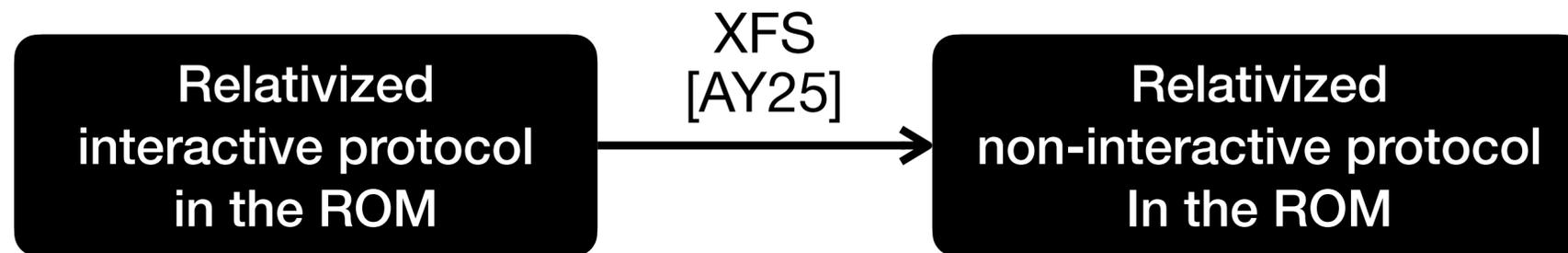


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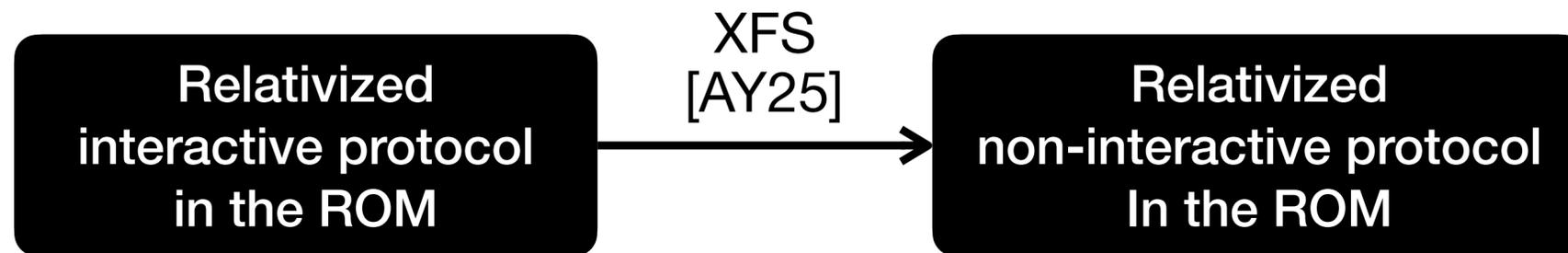
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# Thank you!